

# The Investigation of Geometric Anti-springs Applied to Euler Spring Vibration Isolators

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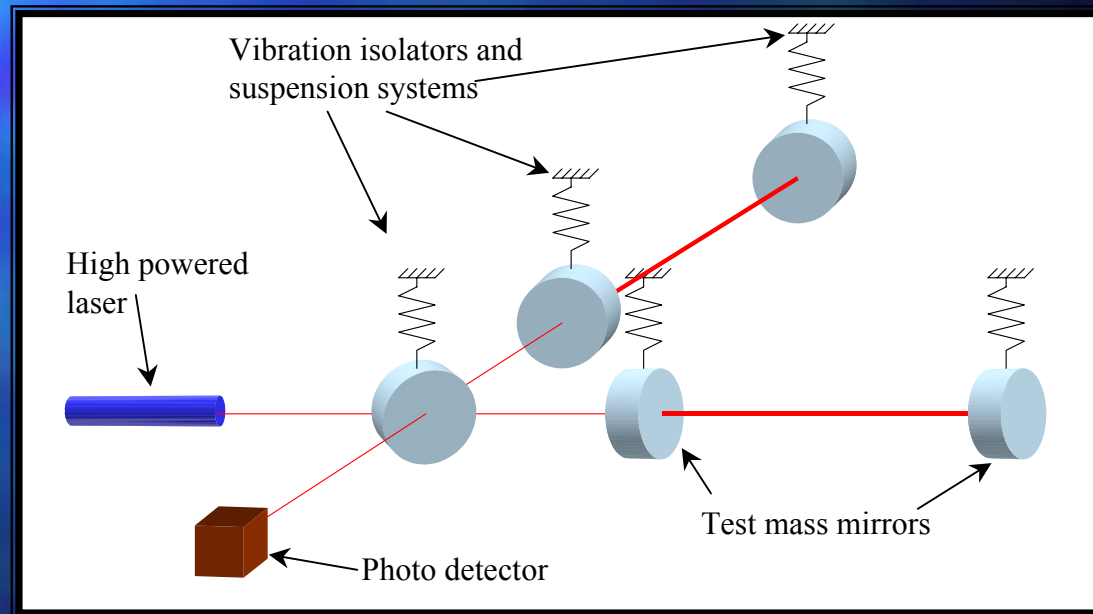
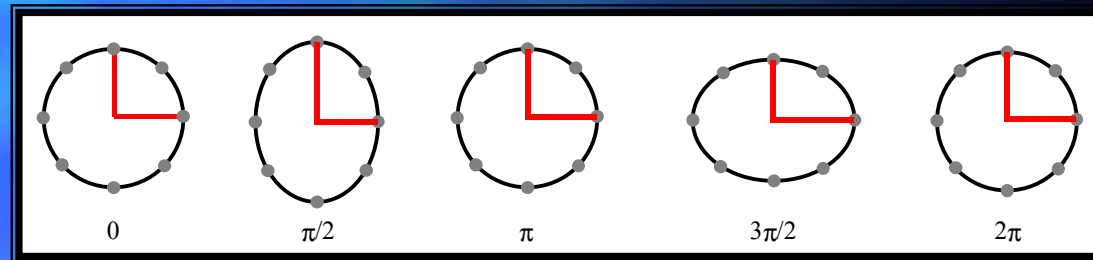
# Presentation Overview

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- Interferometric Gravitational Wave Detectors
- Project Aims
- Experimental Methods
- Creep Problem
- Improvements
- Project Conclusions

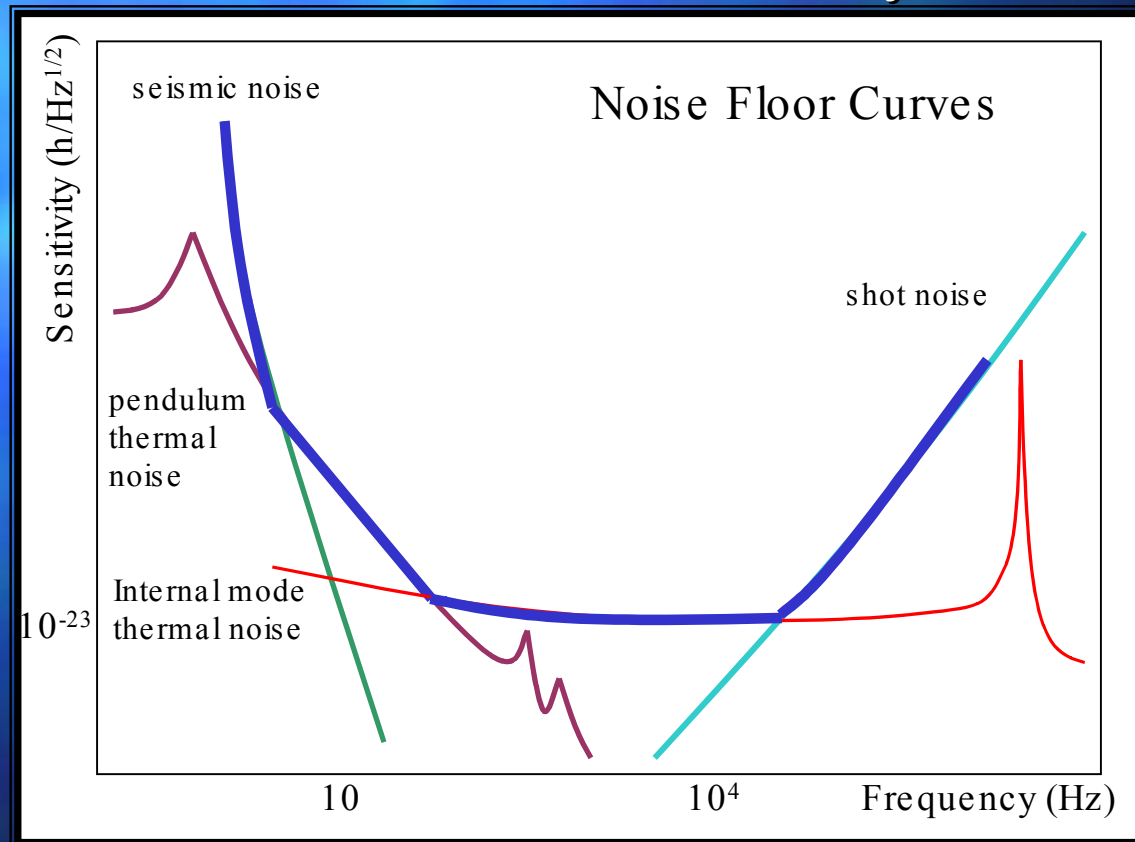
# Interferometric Gravitational Wave Detectors

- Gravitational Waves are ripples in space-time
- Detection can be done based on a Michelson-Morley interferometer



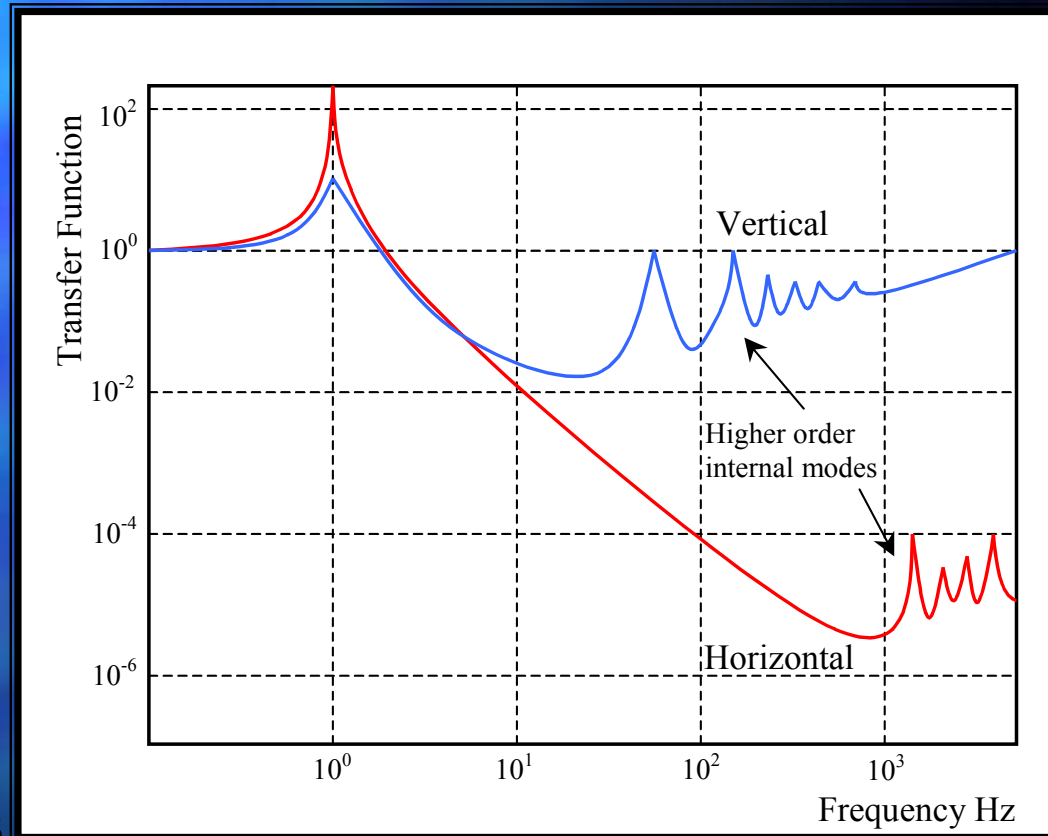
# Noise sources to overcome

- Sensitivity needed of  $h \sim 10^{-22}$  is overcome by noise



# Importance of vibration isolation

- Project concerns reducing seismic noise
- Pendulums are used for horizontal isolation
- Springs are used for vertical
- Attenuation goes as  $1/f^2$  above resonant frequency,  $(f_0 f_1 f_2 \dots f_N / f^N)^2$  for a cascade of N stages



# Project Aims

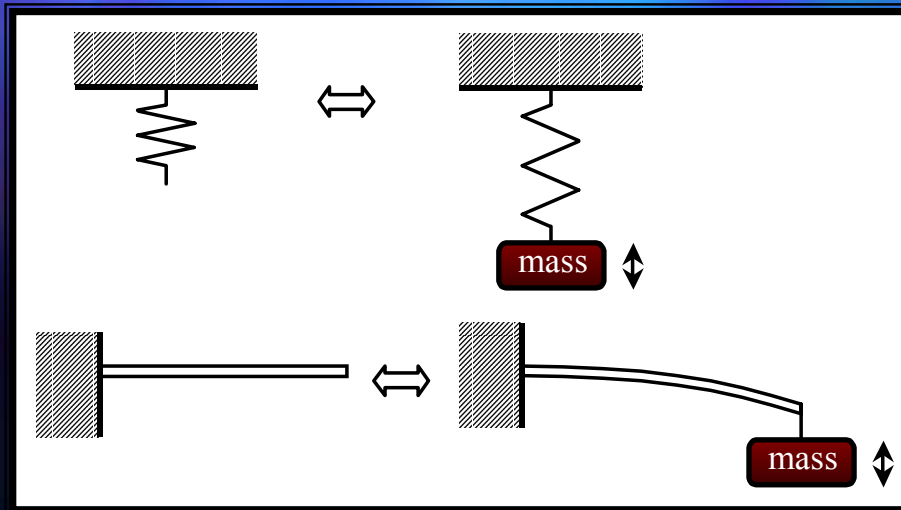
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Concentrates on lowering the resonant frequency of a single vertical vibration isolation stage

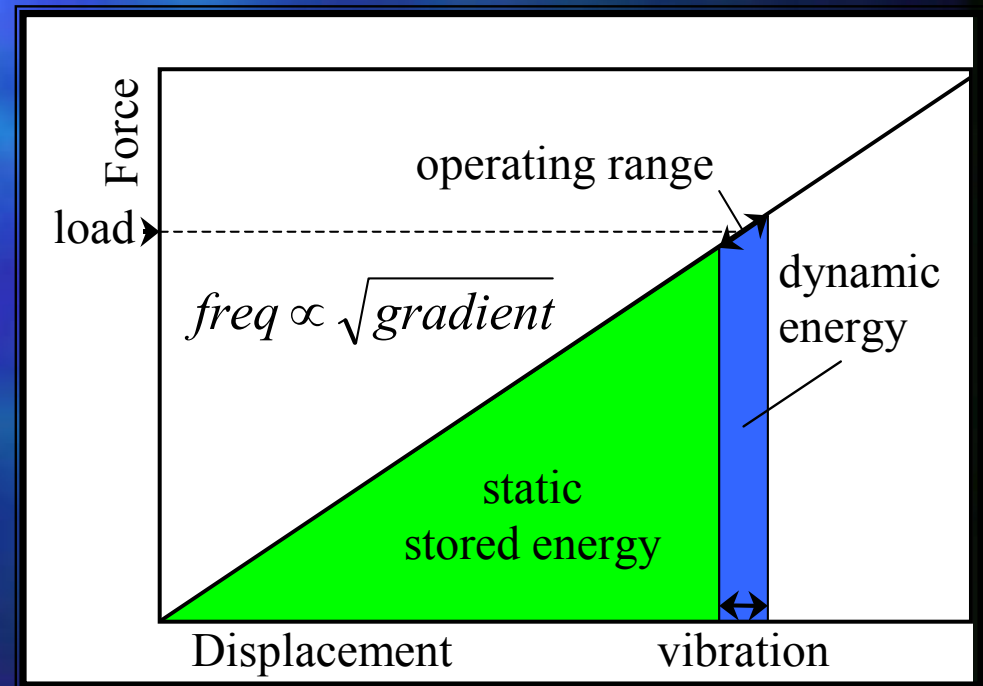
- Euler springs
- Geometric anti-springs
- Euler springs boundary conditions

# Conventional Springs

- Why use Euler springs?
- Conventional springs store large amounts static energy



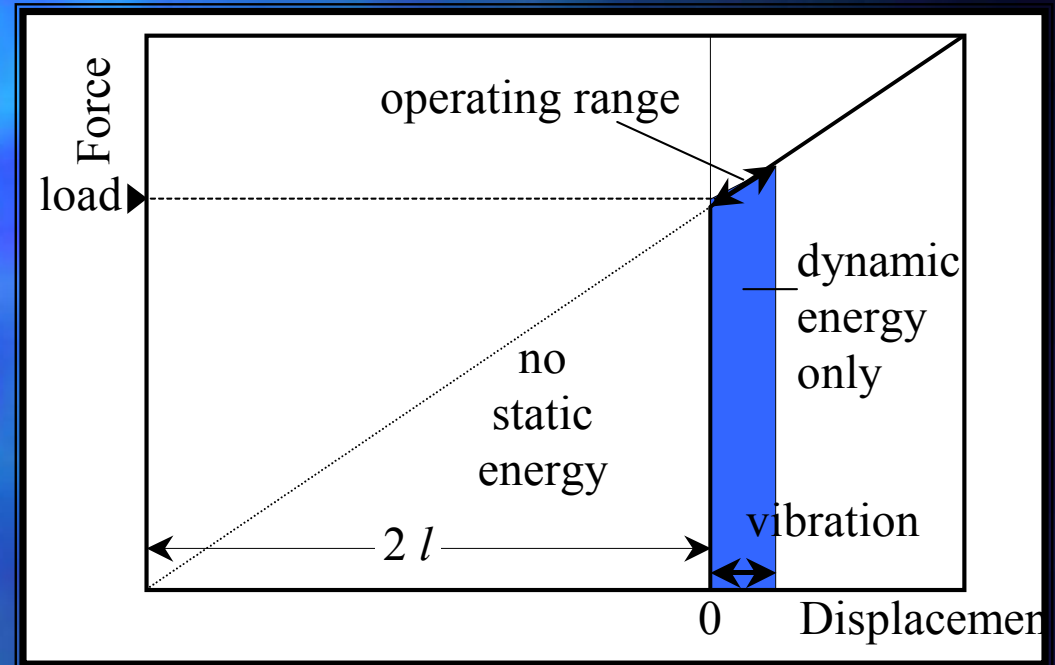
- Stored energy  $\propto$  elastic spring mass required



# Euler Springs

- Stores no static energy  
⇒ low mass
- Resonant frequency is also improved:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{g}{2l}}$$



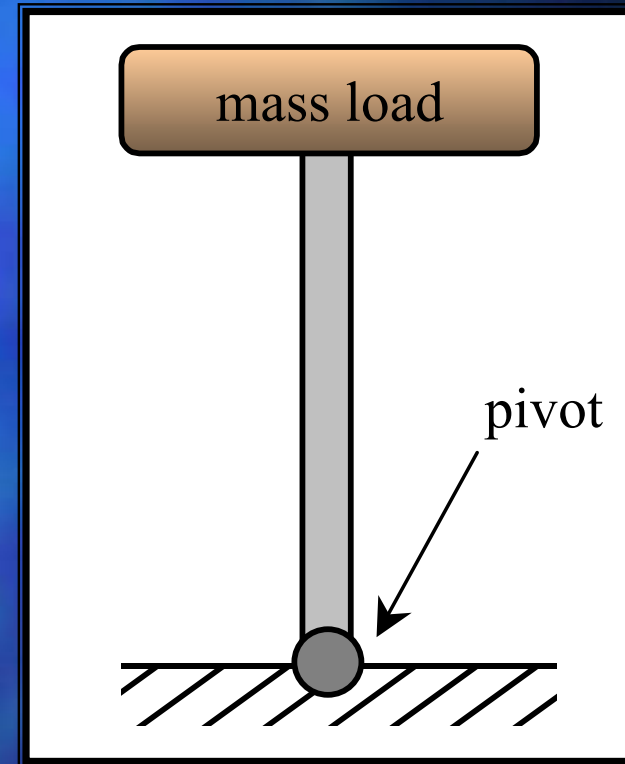
For a conventional spring:

$$\omega = \sqrt{\frac{g}{\Delta l}}$$



# Geometric Anti-Springs

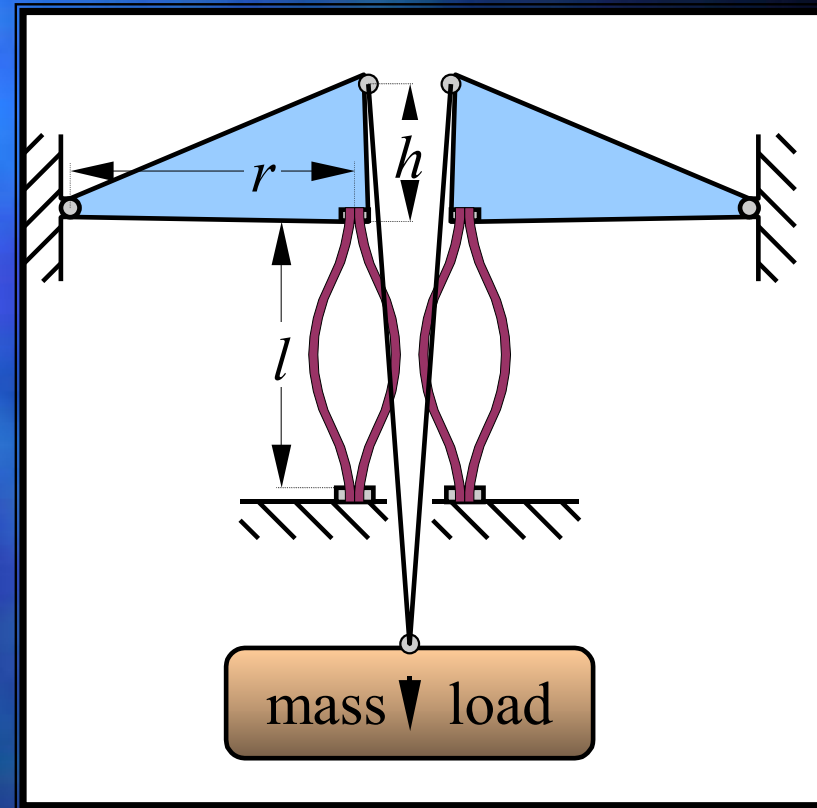
- An anti-spring behaves oppositely to a spring
- The system has a negative spring constant eg. an inverse pendulum



An inverse pendulum

# The Vertical Euler Stage

- We have incorporated the inverse pendulum into the vertical Euler stage
- In effect a lower resonant frequency is resulted



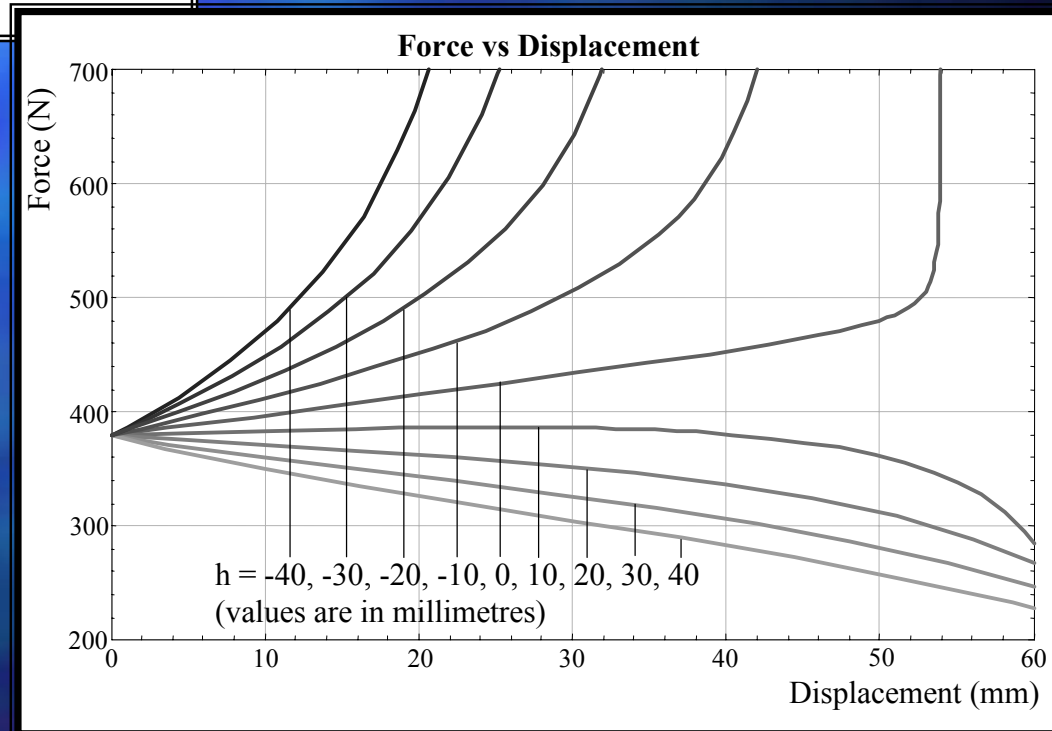
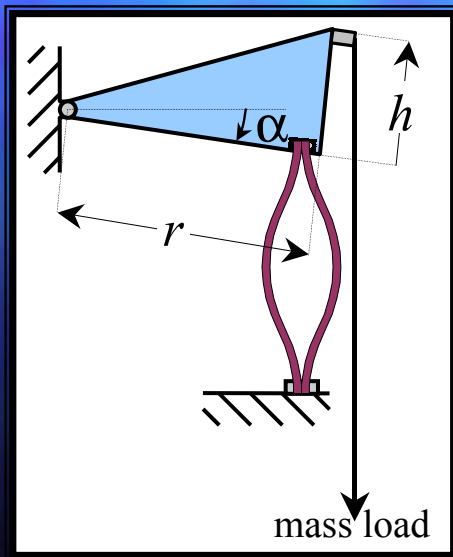
A schematic of the Euler stage

# Mathematical model

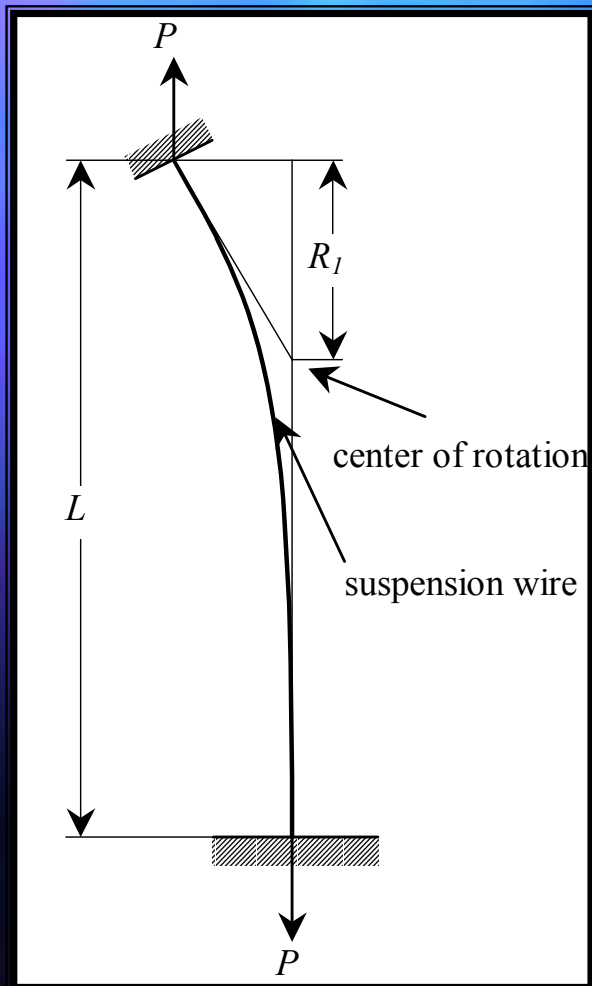
$$Energy(\alpha) = \frac{1}{2} K_w \alpha^2 + \frac{1}{2} \frac{F_{cr}}{2l} r^2 (\sin \alpha)^2 + F_{cr} r \sin \alpha$$

$$y(\alpha) = h - \sqrt{r^2 + h^2} \sin(\arctan(\frac{h}{r}) - \alpha) - 0.01 \sin \alpha$$

$$F_y = \frac{\partial_\alpha Energy(\alpha)}{\partial_\alpha y(\alpha)}$$

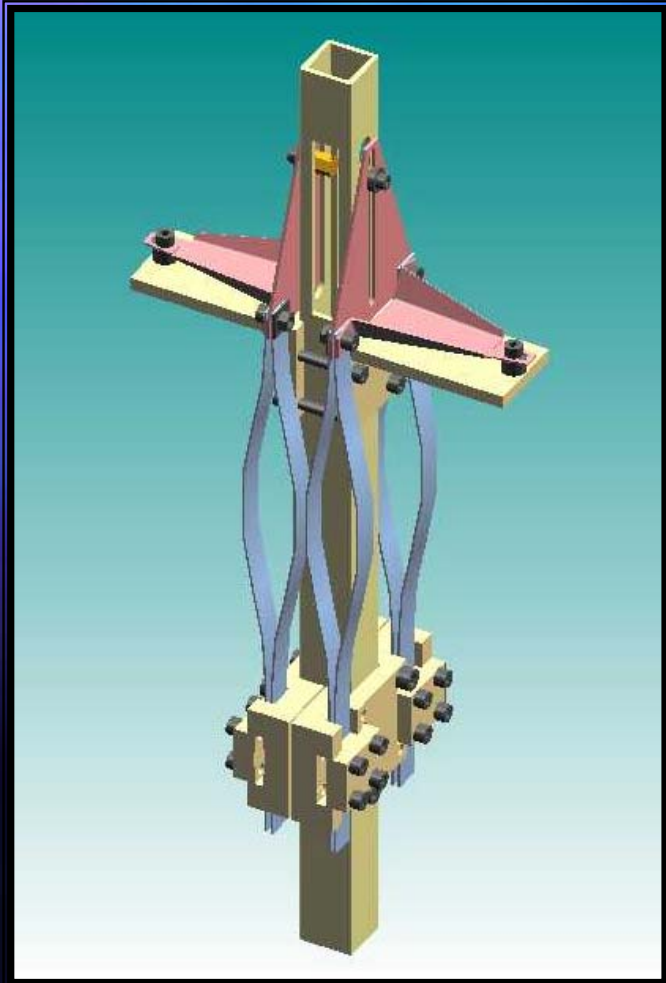


# Suspension wire thickness



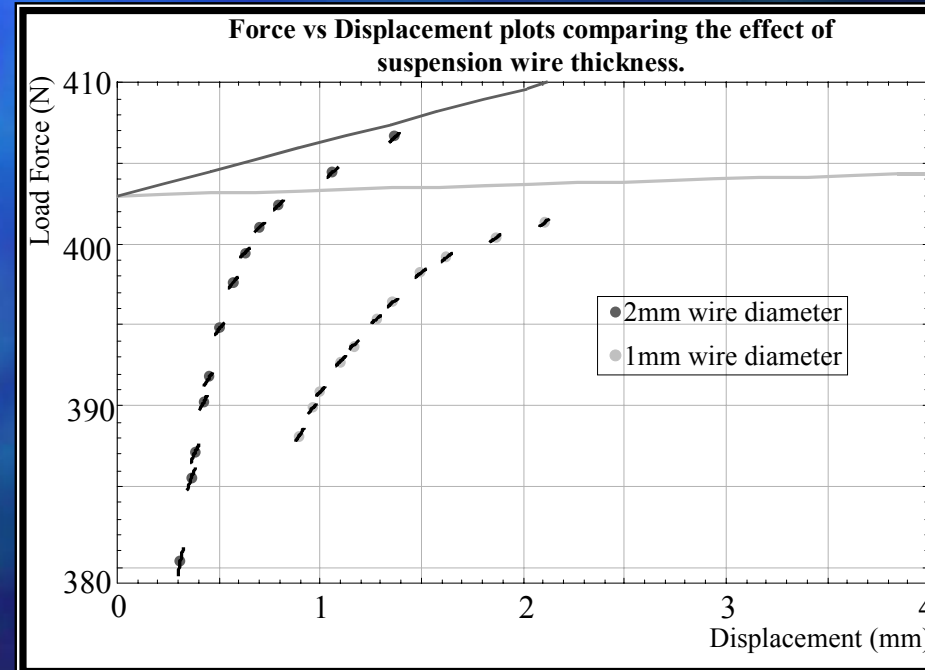
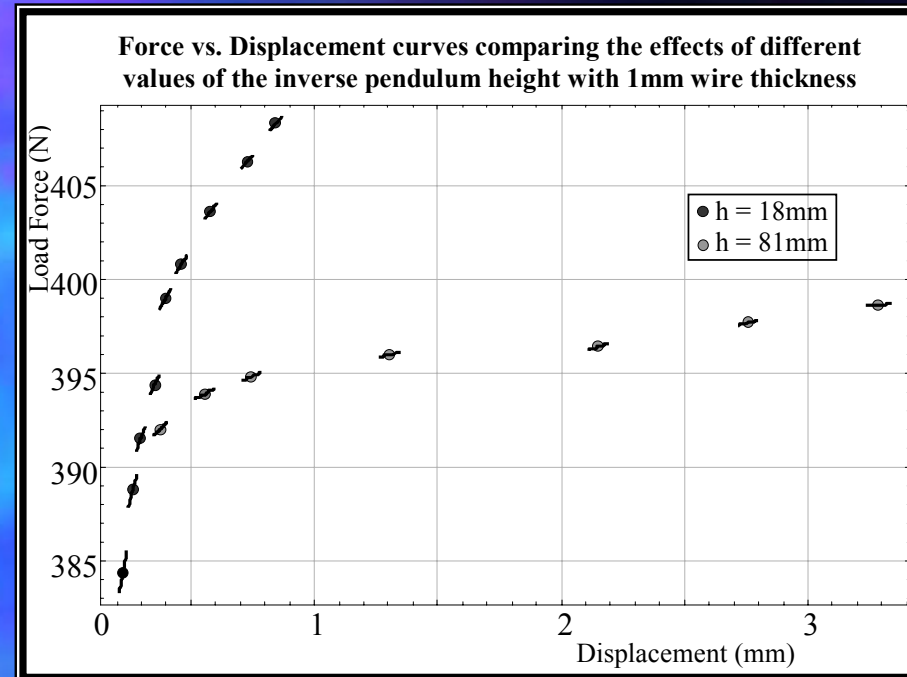
- The performance of the system is greatly effected by the suspension wire
- When the system is in operation it contributes to:
  - an increase stiffness
  - a reduction of the effective  $h$
- Theoretically values of  $R_1$  with a force equal to a test mass ( $\sim 40\text{kg}$ ):
  - 2mm thick wire - 28mm
  - 1mm thick wire - 7mm
  - 0.5mm thick wire - 2mm

# Experimental Set Up



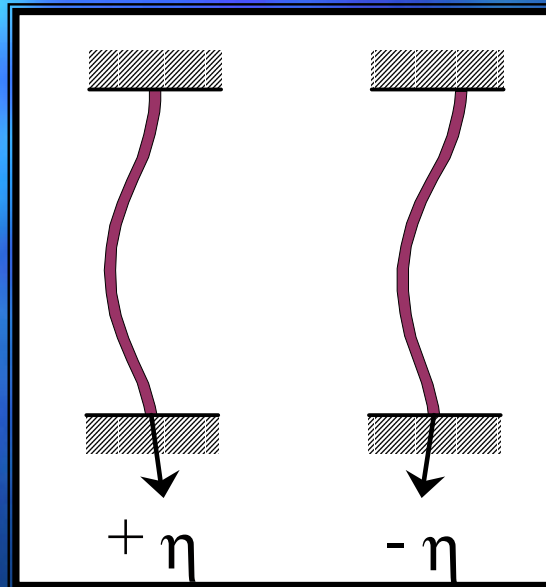
# Results

- 1<sup>st</sup> graph shows strong anti-spring tuning
- A frequency of 0.67Hz with spring constant 720N/m
- 2<sup>nd</sup> graph shows the effect of reducing wire thickness
- Large curvatures can be seen



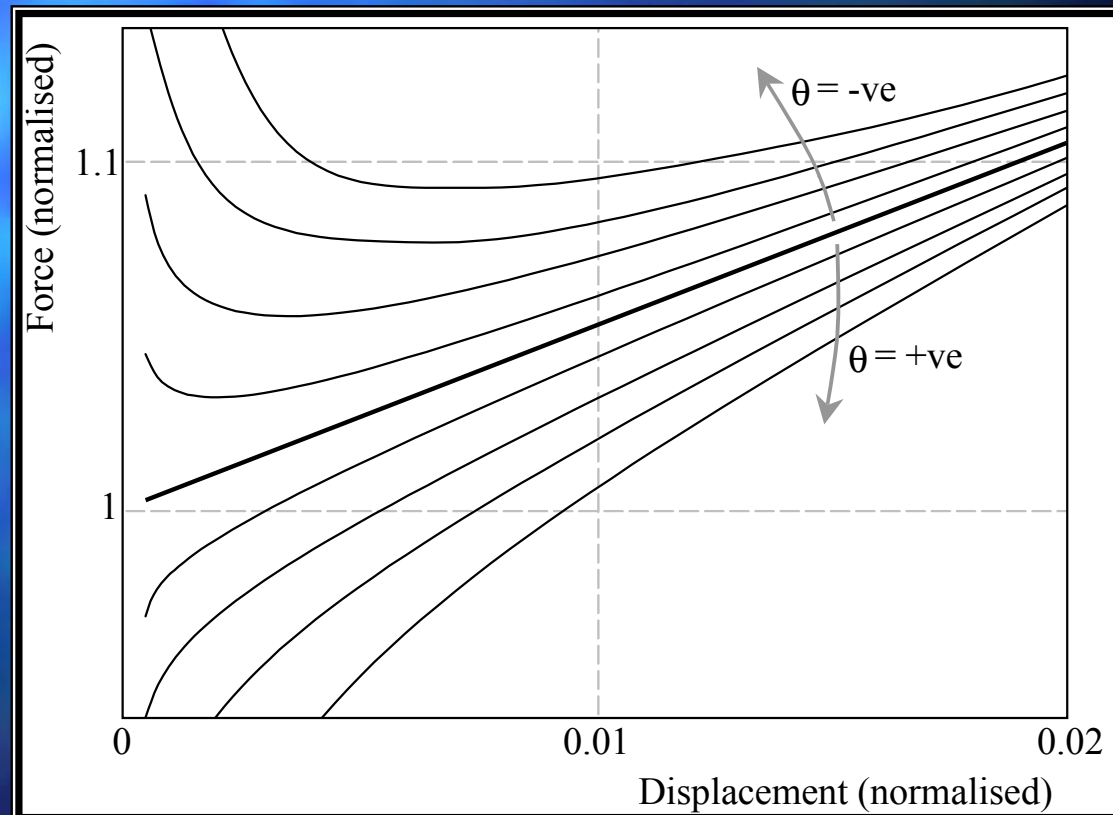
# The investigation of launch angles

- The investigation was motivated by the large curvatures
- It was based on the paper written by Winterflood



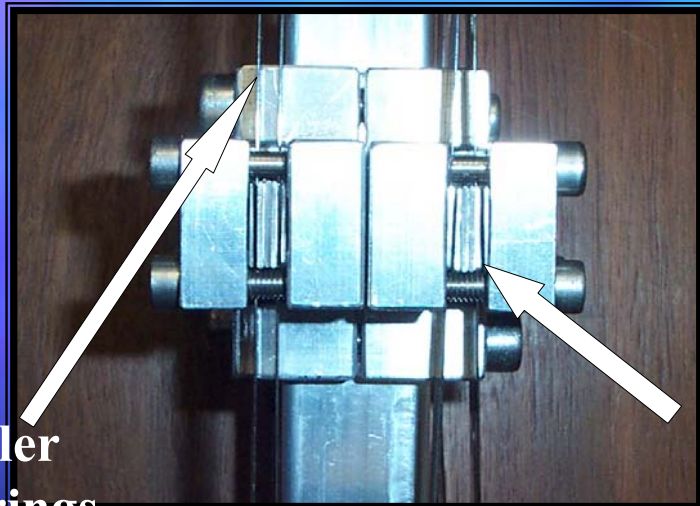
# Euler Launching Angles

- Force-displacement graph shows the principle of varying  $\theta$



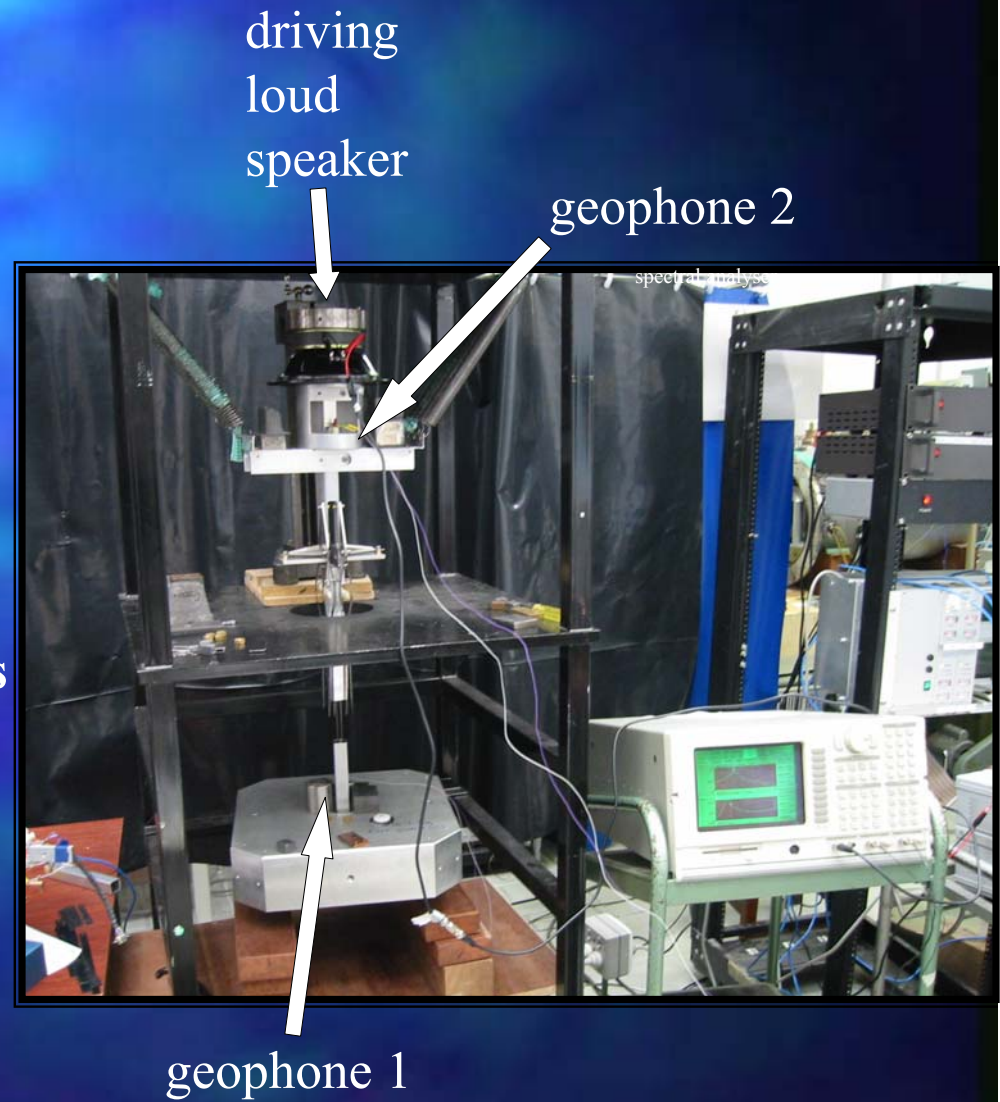
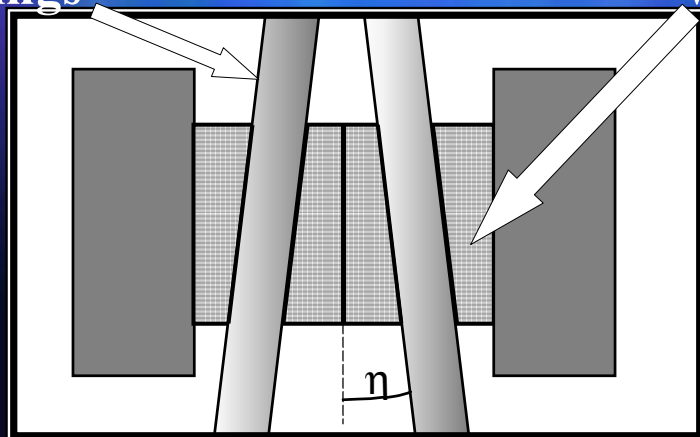


# Experimental Set Up



Euler  
springs

angled  
wedges



driving  
loud  
speaker

geophone 2

geophone 1

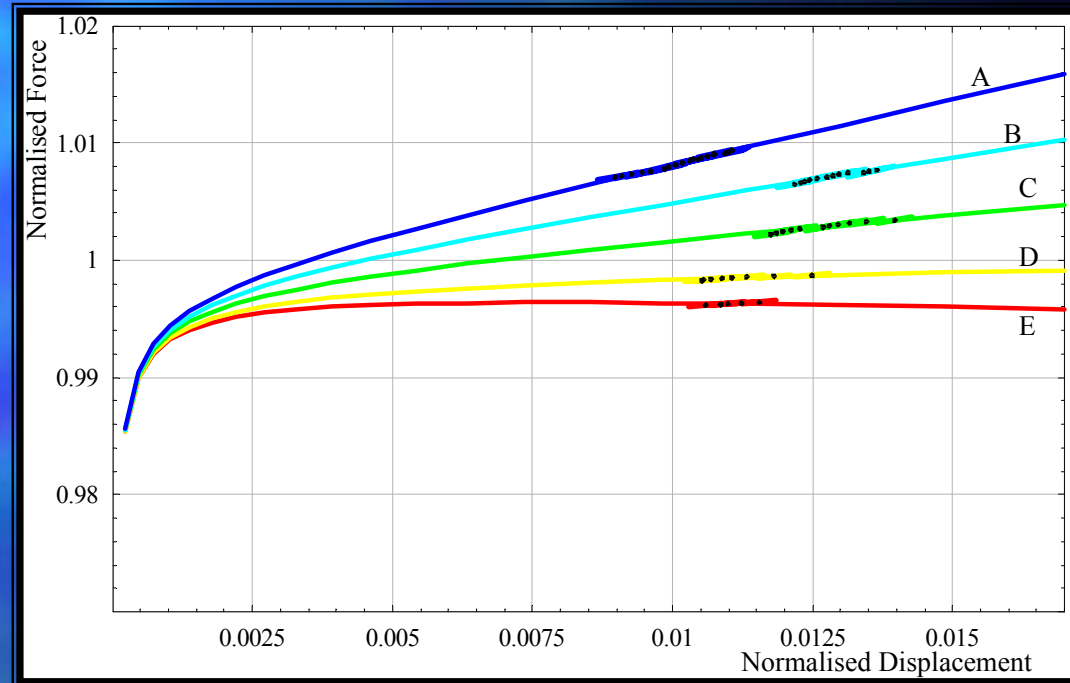
# Experimental Set Up

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- 5 sets of angles were used:
  - a)  $-0.01$  rads
  - b)  $-0.015$ rads
  - c)  $-0.02$ rads
  - d)  $-0.0225$ rads
  - e)  $-0.025$ rads
- Each underwent the inverse pendulum heights of 81, 75, 65, 55 and 45mm:

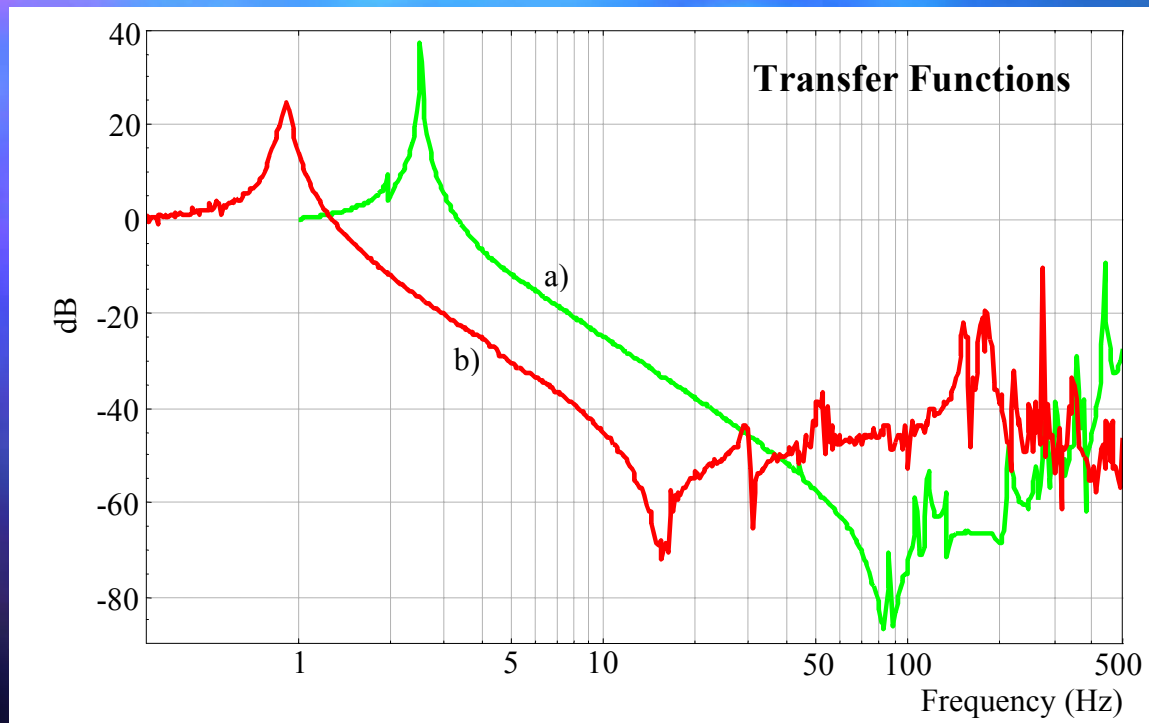
# Results

- Showed large deviations from theory
- Applied best fit curves
- Overall showed general trend
- Explanations:
  - non-ideal clamping conditions
  - yielding of the spring



$\eta = -0.0225$  radians ( $-0.00070$  radians), A)  $h = 45$  mm, B)  $h = 55$  mm, C)  $h = 65$  mm, D)  $75$  mm, and E)  $81$  mm.

# Results

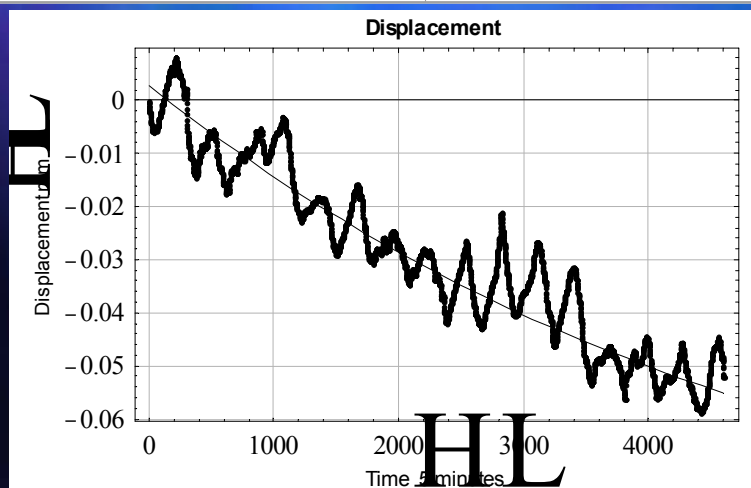


Red shows the transfer function obtained with  $\eta=-0.0225$  with  $h=75\text{mm}$ , and green is the transfer function function obtained in the previous year.

- Transfer function shows  $\sim 20\text{dB}$  improvement
- High noise floor above 20Hz due to centre of percussion effects

# Creep

| Test Material                                     | Temperature Coefficient ( $\mu\text{m}/^\circ\text{C}$ ) | Creep Rate ( $\mu\text{m}/\text{day}$ ) |
|---|--|---|
| AISI C1095 tool steel                             | 16   | 36                                      |
| CS1075 spring steel                               | 4.5  | 14                                      |
| Spring steel with the introduced permanent offset | 6  | ~5 to ~2                                |

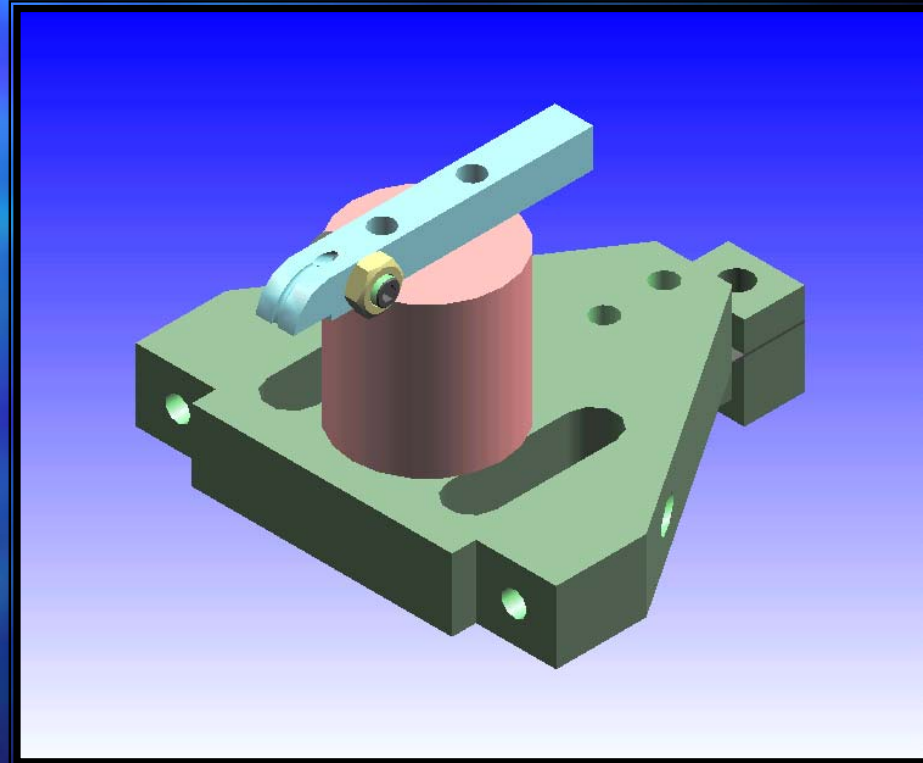
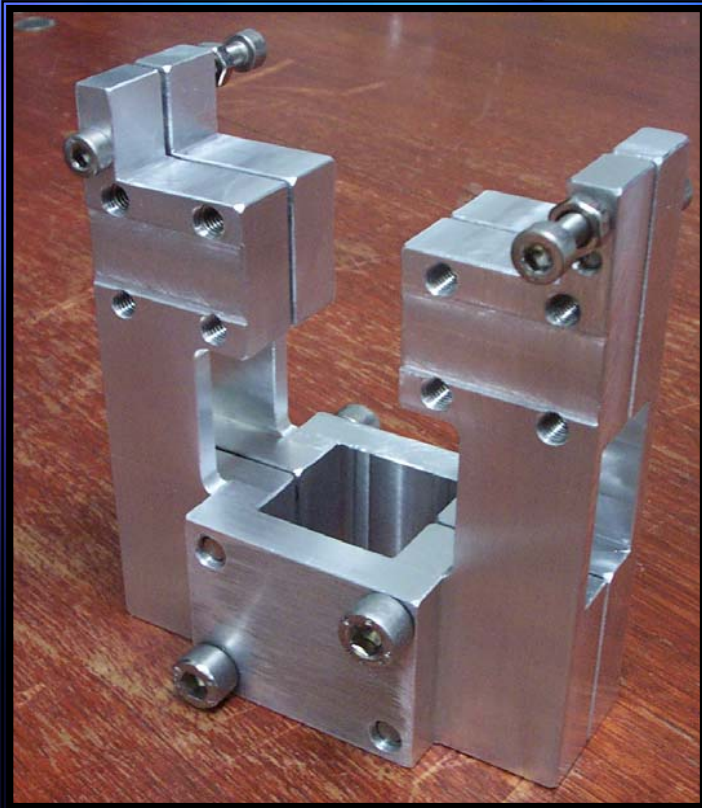


Creep displacement of the third test showing a logarithmic curve

- Creep was constantly experienced during experimentation
- An assessment was made between the current material (AISI C1095 tool steel) and CS1075 spring steel
- Shows that the stability of the Euler experiments can be improved
- Creep rate tends to fit a logarithmic curve

# Design for improvements

- A new angle adjustable clamp
- A new wire holding mechanism for thinner wires



# Project Conclusions

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- A resonant frequency of 0.63Hz was achieved
- Large curvatures in force-displacement plots decreased the performance
- Fine tuning the launch angles proved difficult
- Creeping of the springs was encountered
- Even lower frequencies can be achieved by the design improvements presented